Array-based SMT Encoding for Congestion Control Algorithms

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Problem description

Recently, there has been an increased interest from the research community in using formal methods for analyzing network performance [1, 2]. The proposed techniques are based on SMT solvers (such as Z3 [7]), can model network queues [1] or various congestion control algorithms [2], and can automatically synthesize counter-examples that expose poor performance (e.g., packet loss, low utilization, unfairness). However, they encode continuous models in finite, discrete time, as these representations can be solved more efficiently. These approximations may thus lead to false positives and do not allow one to prove that the desired property holds over larger (potentially infinite) time horizons.

The goal of this project is to design a more generic SMT encoding for congestion control algorithms, which addresses the limitations mentioned above. In particular, we will focus on encoding operations over queues and cumulative functions as operations over arrays, since arrays in SMT-LIB [4] are unbounded and can have indexes of arbitrary types\(^1\).

Example

Let us consider the leaky bucket algorithm, where the packets received by a server are stored in a temporary buffer (“bucket”) and then sent at a constant rate. The cumulative number of received packets is given through the arrival curve \(\alpha(t) = a \cdot t + b\), where \(a\) is the arrival rate and \(b\) is the burst tolerance. Following the ideas from [1, 2], the arrival curve can be encoded in SMT as a sequence of real variables \(\alpha_1, ..., \alpha_T\), where \(T\) is a constant, representing the maximum number of time steps and \(\alpha_i = a \cdot i + b\).

However, one can also use a more generic encoding based on arrays:
\[
\forall t \in \mathbb{R} : \text{select}(\alpha, t) = a \cdot t + b \quad (1)
\]
where \(\alpha\) is an SMT-LIB array with real indexes and real values and \(\text{select}\) is the SMT-LIB function for accessing an element of the array at a given index.

\(^1\)http://smtlib.cs.uiowa.edu/theories-ArraysEx.shtml
Approach

We will start with the SMT encodings generated by the artifact\textsuperscript{2} of [2] for the congestion control algorithms considered in their evaluation: AIMD [6], BBR [5], and Copa [3]. We will then modify the encodings to use arrays (and potentially uninterpreted functions), instead of unrolling the queues and the cumulative functions over time.

Note that our array-based encodings may contain universal quantifiers (as shown in (1)), and quantifier instantiation is generally undecidable. The SMT solver might thus also return unknown (not only sat, unsat, and timeout as before). In such cases, we may also need to augment the encodings with additional terms that trigger particular quantifier instantiations and enable the solver to prove/disprove the satisfiability of the given formula.

At the implementation level, we will extend the artifact to also automatically generate our new encodings and store them in files. We will then automatically run the SMT solver Z3 from command line and integrate its result with the existing visualizer for displaying counter-examples.

Prerequisites

The student is expected to have good programming skills. Prior experience with the SMT-LIB format is a plus.

Opportunities

The student will have the chance to gain a deep understanding of SMT solvers and to learn about congestion control algorithms.

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References


\textsuperscript{2} https://github.com/venkatarun95/ccac


